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THRUST ALLOCATION METHOD FOR OVERACTUATED UNDERWATER ROBOTIC VEHICLE

POVANDENINIO MOBILAUS ROBOTO VARANČIŲJŲ JĖGŲ PASKIRSTYMO METODAS

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The paper describes a method of thrusts allocation in a propulsion system of an underwater robotic vehicle. The vehicle has no other actuators except thrusters so motion and positioning is realised only by change of developed thrusts. A proposed control allocation method has been tested in cases of a fault-free work of the propulsion system and failure of the thruster. A worked out algorithm basis on decomposition of the thruster configuration matrix allows obtaining a minimum Euclidean norm solution. Due to computational simplicity, obtained by applying of singular value decomposition, the proposed approach seems to be an attractive solution for practical applications.

Underwater robot, propulsion system, power distribution.

Introduction

Nowadays, it is common to use underwater robotic vehicles (URVs) to accomplish such missions as inspection of coastal and off-shore structures, cable maintenance, as well as hydrographical and biological surveys. In the military field they are employed in such tasks as surveillance, intelligence gathering, torpedo recovery and mine counter measures.

Their motion of six degrees of freedom (DOF) can be described by the following vectors [1, 3, 4]:

$$\begin{aligned} \boldsymbol{\eta} &= [x, y, z, \phi, \theta, \psi]^T \\ \mathbf{v} &= [u, v, w, p, q, r]^T \\ \boldsymbol{\tau} &= [X, Y, Z, K, M, N]^T \end{aligned} \tag{1}$$

where:

$\boldsymbol{\eta}$ – vector of position and orientation in the inertial frame;

x, y, z – coordinates of position;

ϕ, θ, ψ – coordinates of orientation (Euler angles);

v – the linear and angular velocity vector with coordinates in the body-fixed frame;

u, v, w – linear velocities along longitudinal, transversal and normal axes;

p, q, r – angular velocities about longitudinal, transversal and normal axes;

$\boldsymbol{\tau}$ – vector of forces and moments acting on the robot in the body-fixed frame;

X, Y, Z – forces along longitudinal, transversal and normal axes;

K, M, N – moments about longitudinal, transversal and normal axes.

Nonlinear dynamic equations of motion can be written in a form [3]:

$$\mathrm{M}\dot{\mathrm{v}} + \mathrm{C}(\mathrm{v})\mathrm{v} + \mathrm{D}(\mathrm{v})\mathrm{v} + \mathrm{g}(\eta) = \tau \tag{2}$$

where:

M – inertia matrix (including added mass);

$C(v)$ – matrix of Coriolis and centripetal terms (including added mass);

$D(v)$ – hydrodynamic damping and lift matrix;

$g(\eta)$ – vector of gravitational forces and moments.

The modern URVs are more and more frequently equipped with an automatic control system in order to execute complex manoeuvres without constant human intervention. Basic modules of the control system are depicted in Fig. 1. An autopilot computes demanded propelling forces and moments (commands) $\boldsymbol{\tau}_d$ by comparing desired position and orientation of the robot with their current estimates. Corresponding with them propeller thrusts **f** are calculated in a thrust distribution module and transmitted as control inputs to a propulsion system.

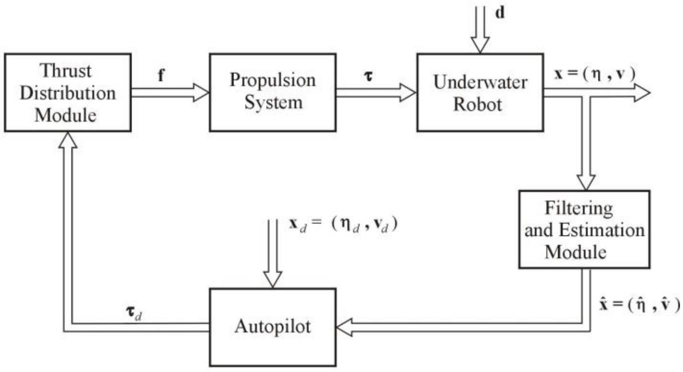

Fig. 1. Structure of automatic control system (**d** –vector of environmental disturbances)

1 pav. Struktūra automatinės valdymo sistemos (d- aplinkos trikdžių vektorius)

The underwater robot regarded in the paper is presented in Fig. 2. It is the open-frame submersible physically connected to the surface by an umbilical cable that provides power and communications. The URV is equipped with a mechanical manipulator needed for underwater interventions.

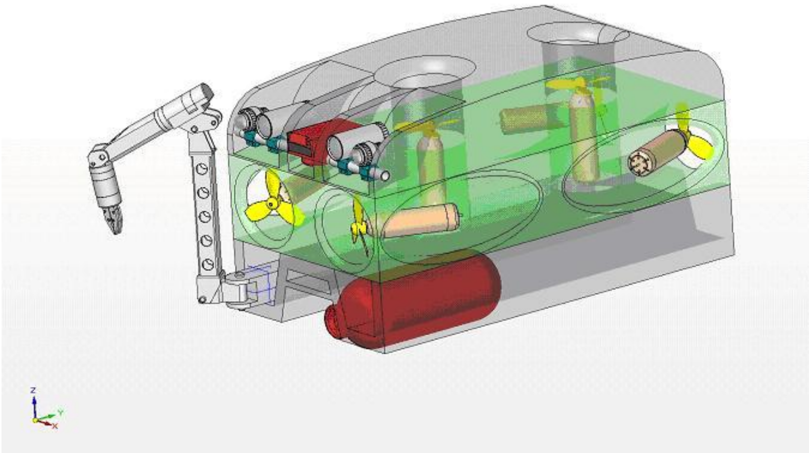

Fig. 2. A virtual view of the URV
2 pav. URV virtualus vaizdas

This underwater apparatus has no other actuators except thrusters so both movement and positioning is realised only by change of thrusts. Its propulsion system consists of six fixed direction thrusters, so the number of actuators is greater than the number of robot's DOF. It means that the propulsion system is designed with redundancy and the robot has capability to maintain position and attitude after any single actuator failure. Hence, an objective of this work is to present a control allocation method both for a fault-free work of the propulsion system and in case of the thruster failure.

Thruster model

A relationship between the desired propelling forces and moments $\boldsymbol{\tau}_d$ and corresponding them thrusts f produced by the propulsion system is a complicated function depending on a density of water, robot's velocity v, actuators diameters and revolutions, etc. (A detailed analysis of thruster's dynamics can be found in e.g. [6]). In many practical applications it is approximated by so called affine model, i.e. a system being linear in its input [3, 7]:

$$\tau_d = \mathrm{Bf} \tag{3}$$

where B is a known constant matrix.

For the URV, presented in Fig. 2, basic motion is displacement in a horizontal plane with some variation due to diving. Since it operates in a crab-wise manner with small roll and pitch angles that can be neglected during normal operations, farther in the paper we will focus only on a plane movement, i.e. motions in surge, sway and yaw. Movement of three DOFs in the horizontal plane is realized by four thrusters assuring speeds up to ±1.2 m/s in longitudinal and ±0.6 m/s in transversal axis. The actuators are mounted askew in relation to main symmetry axes and symmetrically around a robot's centre of gravity (see Fig. 3). Such configuration allows them to produce at the same time not only single propelling forces X and Y or moment N, as shown in Fig. 4, but also any combination of them.

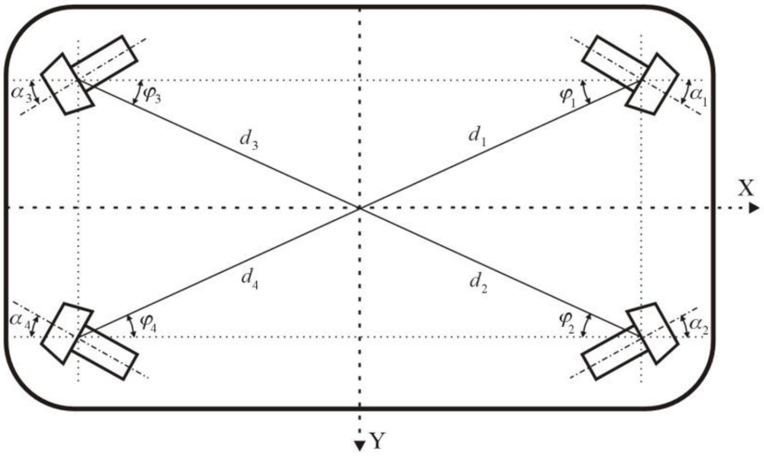

Fig. 3. A configuration of thrusters responsible for horizontal motion (where: d_i – distance of i^{th} thruster from centre of gravity, α_i – angle between longitudinal axis and symmetry axis of ith thruster, φ_i – angle between longitudinal axis and line connecting centre of gravity with centre of symmetry of i^{th} thruster)
3 pav. Varytuvų konfigūracija reikalinga horizontaliam judėjimui

Under assumption that surge, sway and yaw motions are commonly analyzed, the desired vector $\boldsymbol{\tau}_d$ can be described by the following matrix dependence [3, 5]:

$$\tau_d = \mathrm{TPf} \tag{4}$$

where:

$\boldsymbol{\tau}_d = [\tau_{dX}, \tau_{dY}, \tau_{dN}]^T$,

τ_{dX} – force in the longitudinal axis,

τ_{dY} – force in the transversal axis,

τ_{dN} – moment about the normal axis,

$\mathrm{f} = [f_1, f_2, f_3, f_4]^T$,

f_i – thrust of the i^{th} thruster,

T – thruster configuration matrix:

$$\mathbf{T} = \begin{bmatrix} \cos\alpha_1 & \dots & \cos\alpha_4 \\ \sin\alpha_1 & \dots & \sin\alpha_4 \\ d_1 \sin(\alpha_1 - \varphi_1) & \dots & d_4 \sin(\alpha_4 - \varphi_4) \end{bmatrix},$$

P – diagonal matrix of readiness of the thrusters:

$$p_{ii} = \begin{cases} 0 - i^{th} \text{ thruster off} \\ 1 - i^{th} \text{ thruster effective} \end{cases}.$$

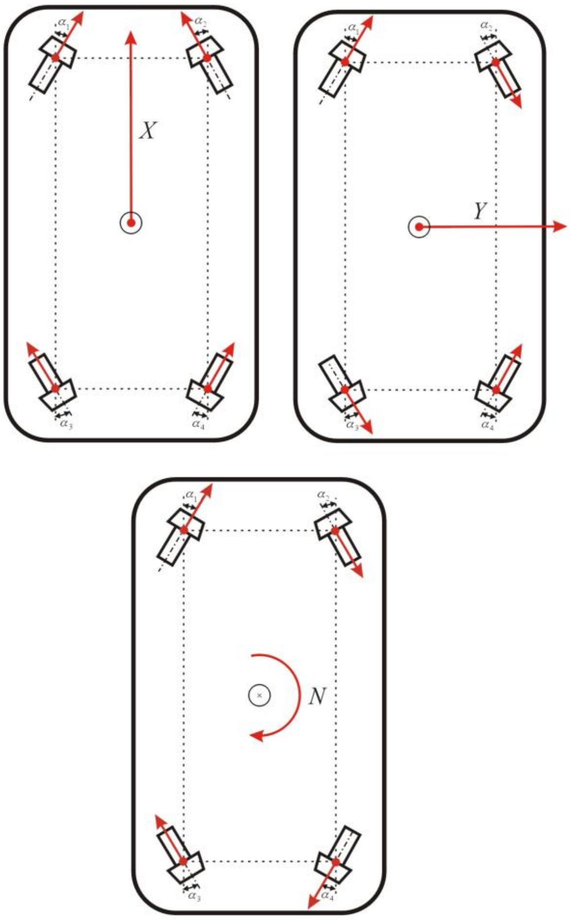

Fig. 4. Allocation for surge, sway and yaw motions

4 pav. Paskirstymas bangavimo svyravimo ir vingiavimo judesiams

Procedure of power distribution

Under assumption that the vector τ_d is bounded, in such a way that the calculated elements of the vector f can never exceed the boundary values f_{min} and $f_{max,}$ the control allocation problem, i.e. computation f from τ_d, is usually formulated as the least-squares optimisation problem:

$$J = \min_{\mathrm{f}} \mathrm{f}^T \mathrm{Hf} \tag{5}$$

subject to:

$$\tau_d - \mathrm{Tf} = 0 \tag{6}$$

where H is a positive definite matrix.

The solution of the above problem with using the Lagrange multipliers is shown in [3] as:

$$\mathrm{f} = \mathrm{T}^* \tau_d \tag{7}$$

where:

$$\mathrm{T}^* = \mathrm{H}^{-1}\mathrm{T}^T \left(\mathrm{TH}^{-1}\mathrm{T}^T\right)^{-1} \tag{8}$$

is the generalized inverse. For the case $\mathrm{H} = \mathrm{I}$ the expression (8) reduces to the Moore-Penrose pseudoinverse [4]:

$$\mathrm{T}^* = \mathrm{T}^T \left(\mathrm{TT}^T\right)^{-1} \tag{9}$$

The above expression is an effective method of finding the optimal allocation for the multi-thruster propulsion system in the fault-free work, but is unsatisfied in case of the actuator failure. In such a case another procedure must be applied assuring that demanded generalised forces and moment can be developed by the remaining actuators. A solution usable in both described cases is proposed below.

Using of singular value decomposition to control allocation

The singular value decomposition (SVD) is an eigenvalue-like decomposition for rectangular matrices [2, 5]. The SVD has the following form for the thrusters configuration matrix T:

$$\mathrm{T} = \mathrm{USV}^T \tag{10}$$

where:
U, V – orthogonal matrices of dimensions 3×3 and 4×4, respectively,

$$\mathrm{S} = \left[\mathrm{S}_T \;\middle|\; 0\right] = \left[\begin{array}{ccc|c} \sigma_1 & 0 & 0 & \\ 0 & \sigma_2 & 0 & 0 \\ 0 & 0 & \sigma_3 & \end{array}\right],$$

$\mathrm{S_T}$ – diagonal matrix of dimensions 3×3,
0– null matrix of dimensions 3×1.

Diagonal entries σ_i are called singular values of the matrix T. They are positive and ordered $\sigma_1 \geq \sigma_2 \geq \sigma_3$.

Decomposition (10) allows working out a computationally convenient procedure to calculate the thrust vector f being a minimum-norm solution to (7). The procedure will be regarded for two cases:

1. all thrusters are operational ($\mathrm{P} = \mathrm{I}$),
2. one of the thrusters is non-operational ($\mathrm{P} \neq \mathrm{I}$).

Algorithm for all thrusters active

Let us denote: $\tau_d = [\tau_{dX}, \tau_{dY}, \tau_{dN}]^T$ – required input vector, $\mathrm{f} = [f_1, f_2, f_3, f_4]^T$ – thrust vector necessary to generate the vector $\boldsymbol{\tau}_d$.

Direct substitution of (10) shows that the vector f determined by formulas (7) and (9) can be written in the form:

$$\mathrm{f} = \mathrm{VS}^* \mathrm{U}^T \tau_d = \mathrm{V} \begin{bmatrix} \mathrm{S}_T^{-1} \\ 0 \end{bmatrix} \mathrm{U}^T \tau_d \tag{11}$$

Algorithm for one thruster non-operational

Let us consider that the k^th thruster is off. It means that $f_k = 0$ and $p_{kk} = 0$. Substitution (10) into the equation (4) leads to the following dependence:

$$\tau_d = \mathrm{TPf} = \mathrm{USV}^T \mathrm{Pf} \tag{12}$$

Defining:
$\mathrm{f}' = [f_1, \ldots, f_{k-1}, f_{k+1}, \ldots f_4]^T$,
$\mathrm{V}^* = \mathrm{V}^T \mathrm{P} = \left[\mathrm{v}_1^*, \ldots, \mathrm{v}_{k-1}^*, 0, \mathrm{v}_{k+1}^*, \ldots, \mathrm{v}_4^*\right]$,

$$V_f^* = \left[v_1^*, ..., v_{k-1}^*, v_{k+1}^*, ..., v_4^*\right] \, ,$$

the expression (12) can be written in a form:

$$\tau_d = USV_f^* f' \tag{13}$$

The matrices U and SV_f^* have dimensions 3×3 so the vector f' can be computed by means of a simple formula:

$$f' = \left(SV_f^*\right)^{-1} U^T \tau_d \tag{14}$$

Hence, the values of the thrust vector f can be obtained as follows:

$$f = [f_1', ..., f'_{k-1}, 0, f_k', ... f_3']^T \tag{15}$$

Simulation study

Computer experiments have been made to confirm validity of the proposed control allocation method under the following assumptions:

1. The nonlinear mathematical model (2) is used to simulate the vehicle's behaviour (see the Appendix),
2. The URV has to follow the desired path beginning from (0 m,0 m), passing target way-points: (0 m, 20 m), (40 m, 35 m), (80 m, 25 m), (80 m, -30 m), (40 m, -40 m), (0m, -20 m and coming back to the start,
3. Its movement is under interaction of environmental disturbances (a sea current with average speed 0.25 m/s and direction 135^0),
4. Hydrodynamic thrusts are computed using formulas (11) or (15) in depends on state of thrusters,
5. Travel time is not fixed, thus the navigation between two way-points is not constrained by time.

Some results of simulations, showing desired and real paths, generalized forces and moment and developed thrusts, are depicted in Fig. 5 and Fig. 6. The first of them is for the failure-free work of the propulsion system and the second for failure of the 3rd thruster. It can be seen that in all cases a path error is almost on the same low level. It indicates that the failure of the actuator has a small influence on accuracy of the robot's motion. The inserted examples demonstrate ability of the proposed control allocation method to cope with a no serviceability of the single actuator in the propulsion system.

Conclusions

The paper presents the method of control allocation for the underwater robot. It can be used for both the fault-free work of the propulsion system and the failure of the thruster.

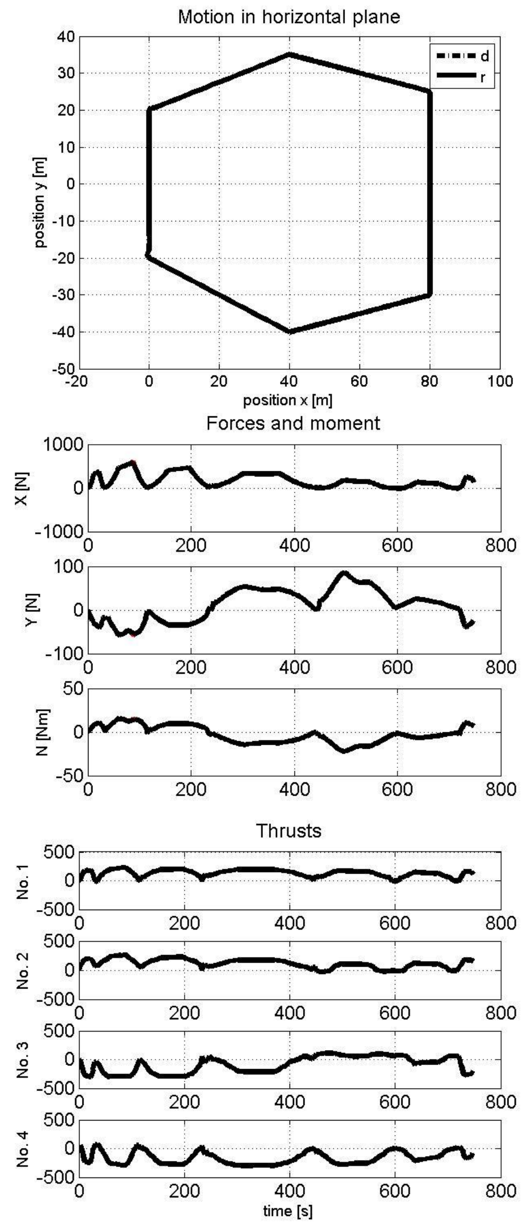

Fig. 5. Simulation results of path-keeping for fault-free case: desired and real path (upper),generalized forces and moment (middle) and developed thrusts (down plot)

5 pav. Palaikymo kelio modeliavimo rezultatai be sutrikimų atvejui: pageidaujamas ir tikras kelias (viršutinis), apibendrintos jėgos ir momentas (viduryje) ir išvystytos varomos jėgos (žemiausiame plane)

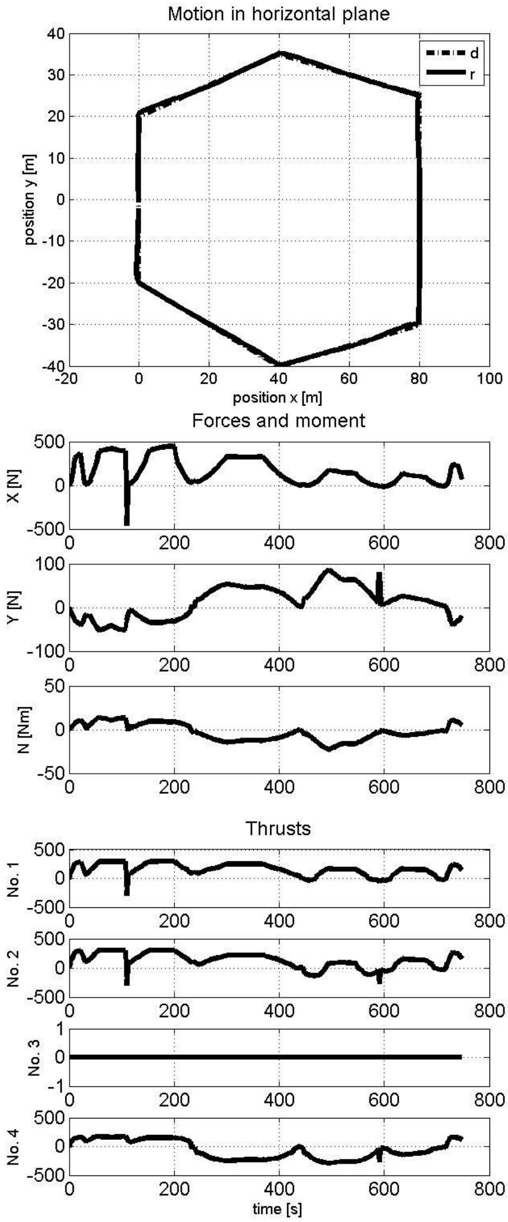

Fig. 6. Simulation results of path-keeping for failure of actuator No. 3: desired and real path (upper), generalized forces and moment (middle) and developed thrusts (down plot)

6 pav. Palaikymo kelio modeliavimo rezultatai pavaros Nr. 3 gedimui: pageidaujamas ir tikras kelias (viršutinis), apibendrintos jėgos ir momentas (viduryje) ir išvystytos varomos jėgos (žemiausiame plane)

The worked out algorithm basis on decomposition of the thruster configuration matrix and allows obtaining minimum Euclidean norm solutions. Due to computational simplicity obtained by applying singular value decomposition the proposed approach can be an attractive alternative to other solutions, e.g. the method using the Lagrange multipliers.

The described control allocation algorithm is of a general character and can be successfully applied to different types of the URVs.

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Appendix

The following model of the URV dynamics was used in the simulation study:

$$\mathrm{M} = diag\{\,99.0,\quad 108.5,\quad 126.5,\quad 8.2\quad 32.9,\quad 29.1\}$$

$$\mathrm{D}(\mathrm{v}) = diag\{10.0,\quad 0.0,\quad 0.0,\quad 0.223\quad 1.918,\quad 1.603\} +$$

$$diag\left\{\begin{matrix} 227.18|u|, & 405.41|v|, & 478.03|w| & \\ & 3.212|p|, & 14.002|q|, & 12.937|r| \end{matrix}\right\}$$

$$C(v)=\begin{bmatrix} 0 & 0 & 0 & 0 & 26.0w & -28.0v \\ 0 & 0 & 0 & -26.0w & 0 & 18.5u \\ 0 & 0 & 0 & 28.0v & -18.5u & 0 \\ 0 & 26.0w & -28.0v & 0 & 5.9r & -6.8q \\ -26.0w & 0 & 18.5u & -5.9r & 0 & 1.3p \\ 28.0v & -18.5u & 0 & 6.8q & -1.3p & 0 \end{bmatrix}$$

$$g(\eta)=\begin{bmatrix} -17.0\sin(\theta) \\ 17.0\cos(\theta)\sin(\phi) \\ 17.0\cos(\theta)\cos(\phi) \\ -279.2\cos(\theta)\sin(\phi) \\ -279.2\left(\sin(\theta)+\cos(\theta)\cos(\phi)\right) \\ 0 \end{bmatrix}$$

The thruster configuration matrix T corresponding to the Fig. 3 was as follows:

$$T=\begin{bmatrix} 0.875 & 0.875 & -0.875 & -0.875 \\ 0.485 & -0.485 & 0.485 & -0.485 \\ 0.332 & -0.332 & -0.332 & 0.332 \end{bmatrix}$$

Jerzy Garus

POVANDENINIO MOBILAUS ROBOTO VARANČIŲJŲ JĖGŲ PASKIRSTYMO METODAS

Reziume

Straipsnyje analizuojamas mobilių povandeninių robotų varančiųjų jėgų paskirstymo metodas. Robotai neturi kitos pavaros, išskyrus stūmimo sistemos varomąją jėgą, todėl judesio ir padėties keitimas vykdomas stūmimo sistemos išvystoma jėga. Pasiūlytas padėties kontrolės metodas buvo išbandytas patikimuose varomosios sistemos darbuose ir jos gedimų atvejais. Algoritmo pagrindu sukurta varomosios sistemos jėgos paskirstymo matrica leidžia gauti minimalų Euklido normos sprendimą. Dėka skaičiavimo paprastumo, taikant išskaidymą po vieną reikšmę, siūlomas sprendimas atrodo patrauklus praktiniam pritaikymui.

Povandeninis robotas, varomoji sistema, galios paskirstymo.

Ержи Гарус

МЕТОД РАСПРЕДЕЛЕНИЯ ВЕДУЩИХ СИЛ МОБИЛЬНОГО ПОДВОДНОГО РОБОТА

Резюме

В статье представлен метод распределения ведущих сил в толкательной системе мобильного подводного робота. Робот не оснащен другими приводами, за исключением толкательной системы, поэтому движение и позиционирование осуществляется только путем изменения направления ведущей силы. Предложенный метод управления распределением сил был протестирован на безотказность работы толкательной установки и выхода из строя двигателя. Разработан алгоритм, на основе разложения матрицы конфигурации двигателя позволяет получить минимальную евклидову норму решения. Благодаря простоте вычислений, предлагаемый подход представляется привлекательным решением для практического применения.

Подводный робот, толкательная система, распределение сил.